Three Dimensional Geometry

Assertion & Reason Type Questions

Directions: In the following questions, each question contains Assertion (A) and Reason (R). Each question has 4 choices (a), (b), (c) and (d) out of which only one is correct. The choices are:

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

c. Assertion (A) is true but Reason (R) is false

d. Assertion (A) is false but Reason (R) is true

Q1. Assertion (A): The points (1, 2, 3), (−2, 3, 4) and (7, 0, 1) are collinear.

Reason (R): If a line makes angles

$\frac{\pi}{2}, \frac{3\pi}{4}$ **and** $\frac{\pi}{4}$ **with**

X, Y and Z-axes respectively, then its direction cosines are 0,

$\frac{-1}{\sqrt{2}}$ **and** $\frac{1}{\sqrt{2}}$.

Answer : (b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

Q2.

Assertion (A): If the cartesian equation of a line is $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$, then its vector form is

$\vec{r}=5\hat{i}-4\hat{j}+6\hat{k}+\lambda(3\hat{i}+7\hat{j}+2\hat{k})$.

Reason (R): The cartesian equation of the line which passes through the point (−2, 4, −5) and parallel to the line given by $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$, is

$\frac{x+3}{-2}=\frac{y-4}{4}=\frac{z+8}{-5}$.

Answer : (c) Assertion (A) is true but Reason (R) is false

Q3.

Assertion (A): The acute angle between the line $\vec{r} = \hat{i} + \hat{j} + 2\hat{k} + \lambda(\hat{i} - \hat{j})$ and the X-axis is $\frac{\pi}{4}$.

Reason (R): The acute angle θ between the lines $\vec{r} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k} + \lambda(a_1\hat{i} + b_1\hat{j} + c_1\hat{k})$ and $\vec{r} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k} + \mu(a_2\hat{i} + b_2\hat{j} + c_2\hat{k})$ is given by $\cos\theta = \frac{|a_1a_2 + b_1b_2 + c_1c_2|}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$.

(CBSE SQP 2022-23)

Answer : (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

Q4.

Assertion (A): The three lines with direction cosines $\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13}; \frac{4}{13}, \frac{12}{13}, \frac{3}{13}; \frac{3}{13}, \frac{-4}{13}, \frac{12}{13}$ are mutually perpendicular.

Reason (R): The line through the points $(1, -1, 2)$ and $(3, 4, -2)$ is perpendicular to the line through the points $(0, 3, 2)$ and $(3, 5, 6)$.

Answer : (b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

Q5.

Assertion (A): The pair of lines given by

$$\vec{r} = \hat{i} - \hat{j} + \lambda(2\hat{i} + \hat{k})$$

and $\vec{r} = 2\hat{i} - \hat{k} + \mu(\hat{i} + \hat{j} - \hat{k})$ intersect.

Reason (R): Two lines intersect each other, if they are not parallel and shortest distance $= 0$.

Answer : (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

Q6.

Consider the lines

$$L_1 : \frac{x+1}{3} = \frac{y+2}{-1} = \frac{z+1}{2}, L_2 : \frac{x-2}{-1} = \frac{y+2}{3} = \frac{z-3}{3}$$

Assertion (A): The lines L_1 and L_2 are mutually perpendicular.

Reason (R): The unit vector perpendicular to both the lines L_1 and L_2 is $\frac{-9\hat{i} - 11\hat{j} + 8\hat{k}}{\sqrt{266}}$.

Answer : (b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

Q7.

Assertion (A): The lines $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu \vec{b_2}$ are perpendicular, when $\vec{b_1} \cdot \vec{b_2} = 0$.

Reason (R): The angle θ between the lines $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu \vec{b_2}$ is given by $\cos\theta = \frac{\vec{b_1} \cdot \vec{b_2}}{|\vec{b_1}||\vec{b_2}|}$.

(CBSE 2023)

Answer : (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

Assertion (A): $x^2 + y^2 + z^2 + 4x - 6y - 8z = 7$ the equation to the sphere whose centre is at (–2, 3, 4) and radius is 6 units.

Reason (R): Given:

Centre is at (–2, 3, 4) and $r = 6$

$\Rightarrow (x_0, y_0, z_0) = (-2, 3, 4)$ and $r = 6$

We know that general equation of sphere is

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$$

$$\Rightarrow \quad (x - (-2))^2 + (y - 3)^2 + (z - 4)^2 = 6^2$$

$$\Rightarrow \quad (x + 2)^2 + (y - 3)^2 + (z - 4)^2 = 6^2$$

$$\Rightarrow \quad x^2 + 4x + 4 + y^2 - 6y + 9 + z^2 - 8z + 16 = 36$$

$$\Rightarrow \quad x^2 + y^2 + z^2 + 4x - 6y - 8z + 29 = 36$$

$$\Rightarrow \quad x^2 + y^2 + z^2 + 4x - 6y - 8z = 7$$

Ans. Option (A) is correct.

> ***Explanation:*** Assertion (A) and Reason (R) both are correct and Reason (R) is the correct explanation of Assertion (A).

Assertion (A): If two lines are in the same plane *i.e.*, they are coplanar, they will intersect each other if they are non-parallel. Hence the shortest distance between them is zero.

If the lines are parallel then the shortest distance between them will be the perpendicular distance between the lines *i.e.*, the length of the perpendicular drawn from a point on one line onto the other line.

Reason (R): The angle between the lines with direction ratio $\langle a_1, b_1, c_1\rangle$ and $\langle a_2, b_2, c_2\rangle$ is given by:

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

Ans. Option (B) is correct.

Explanation: Assertion (A) and Reason (R) both are individually correct.

Assertion (A): Direction cosines of a line are the sines of the angles made by the line with the negative directions of the coordinate axes.

Reason (R): The acute angle between the lines $x - 2 = 0$ and $\sqrt{3x - y - 2}$ is 30°.

Ans. Option (D) is correct.

Explanation: Assertion (A) is wrong.

Since, direction cosines of a line are the cosines of the angles made by the line with the positive directions of the coordinate axes.

Reason (R) is correct.

Since, the slope of the line $x - 2 = 0$ is ∞.

The slope of line $\sqrt{3}x - y - 2 = 0$ is $\sqrt{3}$.

Let $m_1 = \infty$, $m_2 = \sqrt{3}$ and the angle between the given lines is θ.

$$\Rightarrow \quad \tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 \cdot m_2}\right|$$

$$\Rightarrow \quad \tan\theta = \left|\frac{\frac{m_2}{m_1} - 1}{\frac{1}{m_1} + m_2}\right|$$

$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \theta = 30°$$

Assertion (A): *P* is a point on the line segment joining the points (3, 2, –1) and (6, –4, –2). If *x* coordinate of *P* is 5, then its *y* coordinate is –2.

Reason (R): The two lines $x = ay + b, z = cy + d$ and $x = a'y + b', z = c'y + d'$ will be perpendicular, iff $aa' + bb' + cc' = 0$.

Ans. Option (C) is correct.

Explanation: Assertion (A) is correct.

Since $P = (5, y, z)$

Equation of line joining (3, 2, –1) and (6, –4, –2) is

$$\frac{x-3}{6-3} = \frac{y-2}{-4-2} = \frac{z+1}{-2+1} = \frac{x-3}{3} = \frac{y-2}{-6} = \frac{z+1}{-1}$$

so if point *P* lies on the line then it must satisfy the above equation

$$\frac{5-3}{3} = \frac{y-2}{-6} = \frac{z+1}{-1}$$

$$\frac{5-3}{3} = \frac{y-2}{-6}$$

Hence *y* co-ordinate of P is –2.

Reason (R) is false.

Since, the two lines $x = ay + b, z = cy + d$ and $= a'y + b', z = c'y + d'$ will be perpendicular, iff $aa' + cc' + 1 = 0$.

Assertion (A): The angle between the straight lines $\frac{x+1}{2} = \frac{y-2}{5} = \frac{z+3}{4}$ and $\frac{x-1}{1} + \frac{y+2}{2} = \frac{z-3}{-3}$ is 90°

Reason (R): Skew lines are lines in different planes which are parallel and intersecting.

Ans. Option (C) is correct.

Explanation: Assertion (A) is correct.

Given : $\frac{x+1}{2} = \frac{y-2}{5} = \frac{z+3}{4}$

and $\frac{x-1}{1} + \frac{y+2}{2} = \frac{z-3}{-3}$

Direction ratios of lines are $a_1 = 2, b_1 = 5, c_1 = 4$ and $a_2 - 1, b_2 - 2, c_2 - -3$

As we know, The angle between the lines is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\left(\sqrt{a_1^2+b_1^2+c_1^2}\right).\left(\sqrt{a_2^2+b_2^2+c_2^2}\right)}$$

$$\Rightarrow \quad \cos\theta = \frac{2\times1+5\times2+4\times-3}{\left(\sqrt{2^2+5^2+4^2}\right).\left(\sqrt{1^2+2^2+(-3)^3}\right)}$$

$$= 0$$

$$\therefore \quad \theta = 90°$$

Reason (R) is wrong.

In the space, there are lines neither intersecting nor parallel, such pairs of lines are non-coplanar and are called skew lines.

Assertion (A): The length of the intercepts on the co-ordinate axes made by the plane $5x + 2y + z - 13 = 0$ are $\frac{13}{5}, \frac{13}{2}, 13$ unit

Reason (R): Given:

Equation of plane

$$5x + 2y + z - 13 = 0$$

$$\Rightarrow \quad 5x + 2y + z = 13$$

$$\Rightarrow \quad \frac{5x + 2y + z}{13} = 1$$

$$\Rightarrow \quad \frac{x}{\frac{13}{5}} + \frac{y}{\frac{13}{2}} + \frac{z}{\frac{13}{1}} = 1$$

∴ Length of intercepts are $\frac{13}{5}, \frac{13}{2}, 13$ units

Ans. Option (A) is correct.

Explanation: Assertion (A) and Reason (R) both are correct and Reason (R) is the correct explanation of Assertion (A).